Associations Between Student Pursuit of Novel Mathematical Ideas and Resilience

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As part of a broader study of characteristics of situations that promote or inhibit spontaneous student exploration of novel mathematical ideas, student inclination to display these spontaneous behaviours was studied in conjunction with resilience. Resilience was operationalised using the dimensions of optimism (Seligman, 1995). Indicators of optimism were identified in post-lesson video-stimulated reconstructive interviews with year eight students. Students who demonstrated the pursuit of novel mathematical ideas were found to also display indicators of resilience.

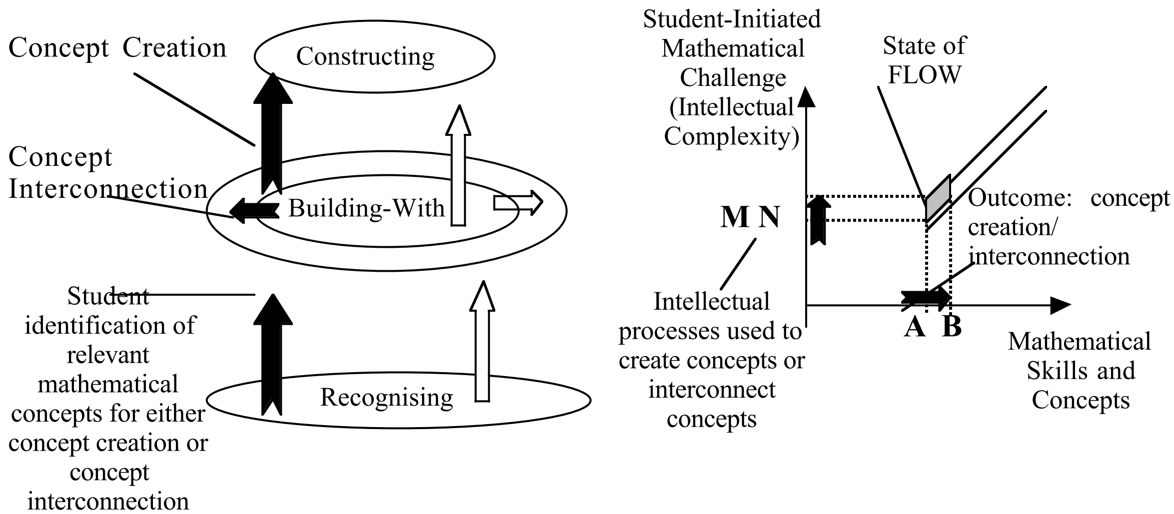
This research interconnects "adolescent cognition, problem solving behaviours, and resilience in contexts of adversity"; the type of interdisciplinary study identified by (Galambos & Leadbeater, 2000, p. 289) as essential for furthering our understanding of young people. Resilience, which can be described in everyday language as being able to rebound or show recuperative power, has been variously described in the literature as (a) '... the mechanisms and processes that lead some individuals to thrive despite adverse life circumstances.' (Galambos & Leadbeater, 2000, p. 291); (b) "… academic and emotional and social competence despite adversity and stress." (Nettles, Mucherah, & Jones, 2000, p. 47); and (c) an "optimistic orientation" to the world characterized by a positive explanatory style where successes are perceived as permanent, pervasive, and personal, and failures as temporary, specific, and external (Seligman, 1995). The context of adversity in the present study is characterized by the process mathematical problem solving; the pursuit of many unproductive pathways in search of productive pathways (often many 'mathematical failures' before a success). This study explores associations between student resilience and student inclination to explore novel mathematical ideas using Seligman's dimensions of optimism to operationalisation resilience. Video-stimulated post-lesson interviews with students were used to identify indicators of resilience. The findings from previous research about the cognitive activities undertaken by these students informed this study (Williams, 2001; Williams, submitted). It was found that students who demonstrated the inclination to pursue novel mathematical ideas also displayed indicators of resilience. This study illuminates a possible aspect of the correlation found by Yates (2002) between optimism and general mathematical achievement.

Literature Review and Theoretical Framework

Student-initiated and student directed cognitive activity is not generally found in traditional classrooms where the teacher demonstrates the mathematical procedures the students are required to undertake. In these instances, the student is not required to analyse the task and identify for themselves a sequence of known procedures to formulate their own solution pathway to solve the problem ("concept interconnection"). Even in classrooms of a more exploratory nature where a more student-centred approach is encouraged and students frequently undertake concept interconnection, students rarely encounter tasks that promote the synthesis and evaluation of mathematical ideas and concepts to form novel mathematical concepts ("concept creation") (Wood, Williams & McNeal, submitted).

Key:

Part of the spontaneous process of concept interconnection or concept creation (discovering complexity)



(a) Dreyfus, Hershkowitz, and Schwarz's elements of the process of abstraction.

(b) Conditions for Csikszentmihalyi's flow (1992).

Figure 1. Williams' diagrammatic representations of students discovering complexity (the processes of concept creation and/or concept interconnection) highlighting: (a) Dreyfus, Hershkowitz and Schwarz' cognitive elements of the process of abstraction; and (b) Csikszentmihalyi's conditions for flow.

Figure 1 provides two different and consistent representations of the processes related to student development of novel ideas. Figure 1 Diagram (a) is a visual representation of Dreyfus, Hershkowitz, & Schwarz's (2001) nested observable cognitive elements of the process of abstraction; and Diagram (b) is a visual representation of the same student-initiated process from the perspective of the conditions for flow. Detailed descriptions of these two representations have been provided in previous papers (Williams, 2002b, 2002a). The black arrows in Figure 1 identify the elements of the process of abstraction that involve students spontaneously 'moving into the unknown' or pursuing mathematical ideas they formulate for themselves rather than replicating procedures explained by the teacher. Diagram (a) differentiates between the two types of cognitive activity upon which this study focuses (concept interconnection and concept creation). Concept interconnection (represented by the black vertical arrow from recognising to building-with in conjunction with the black horizontal arrow in the building-with region) involves student selection and sequencing of appropriate mathematical procedures to solve an unfamiliar problem. The vertical arrow represents the student selecting or assembling appropriate mathematical ideas and procedures ready for use, and the horizontal arrow represents the applying of each known procedure selected (inner circle) in an unfamiliar sequence (outer circle) in a new context. This process strengthens recognition of, and the capability to apply mathematical procedures in various contexts. Concept creation (represented by the two vertical black arrows in Diagram (a)) refers to constructing nested within building-with and recognising. Students undertaking this process select, integrate, and synthesise mathematical ideas and concepts to create novel mathematical concepts. Examples of these processes can be found in Williams (2001, 2002a, submitted) and Dreyfus, Hershkowitz, and Schwarz (2001). The non-black arrows on the right in Figure 1, Diagram (a) represent a teacher-directed rather than student-initiated and student-directed abstraction process.

Figure 1 Diagram (b), represents the same spontaneous student processes represented by the black arrows in Diagram (a) but in this case, the representation breaks the spontaneous student processes into the conditions for "flow" (optimal learning conditions) specific to solving unfamiliar challenging problems in mathematics which has been identified as "discovered complexity" (Williams, 2000). Discovered complexity is characterised by a spontaneous student question that requires the use of complex thinking to resolve the question (trying to meet the intellectual challenge of moving from **M** to **N** on the vertical axis) and results in concept creation or concept interconnection (increase in mathematical skills and concepts from **A** to **B** on the horizontal axis). A person working above their present skill level on a challenge almost out of reach meets Csikszentmihalyi's (1992) conditions for flow. The state of flow (represented by a coordinate in the shaded region of flow on Figure 1, Diagram (b)) is characterised by students losing all sense of self, time, and the world around because all energies are directed to the task at hand. The intellectual challenge within the process of concept interconnection involves analysis, synthetic-analysis, evaluative-analysis whilst the process of concept creation involves the cognitive processes employed in concept interconnection in conjunction with synthesis and evaluation. The visual representation in Figure 1 Diagram (b) does not differentiate between concept creation and concept interconnection; these differences are hidden within the complexity of intellectual challenge (on the vertical axis) and the nature of the mathematical outcome (on the horizontal axis). A more detailed explanation of the flow diagram can be found in a previous paper (Williams, 2002a).

The question remains: Why are some students inclined to challenge themselves to spontaneously explore novel mathematical ideas and other students inclined to stay within the confines of the mathematical ideas presented and explained by the teacher? Seligman (1995) used Csikszentmihalyi's (1992) concept of flow as a framework to discuss how an optimistic orientation to the world can be developed in young children to increase their resilience (Seligman et al., 1995). He found resilience could be developed by engineering the conditions for flow; in overcoming small challenges to gain successes, the child's inclination to undertake future challenges was increased. Optimistic children perceived good fortune to result from their own endeavours rather than occur as a matter of chance, they saw failures as temporary and as able to be overcome by their own endeavours. They also generalised successes as personal attributes and constrained failures to the specific situations in which the failure occurred. Table 1 summarises Seligman's dimensions of optimism which are used as indicators of resilience in the present study.

Numerous variables previously studied (success, persistence, engagement, confidence, perceived usefulness of mathematics, choice of challenging task, mathematics anxiety, learned helplessness, causal attribution style, ability, effort, task difficulty, and luck) (e.g., Fennema & Leder, 1993; Yates, 2002) fit in interesting ways within the framework in Figure 1. Engagement is a property of the state of flow, and success is generally an outcome of flow. When students have the opportunity to create or interconnect concepts (discover complexity), task difficulty, choice of challenging task, and usefulness of the mathematics fall within the domain of student choice rather than the domain of teacher imposition. Mathematical anxiety and learned helplessness (Seligman, 1995), have been found to exist where the task is too challenging or the student is required to use skills too far above their perceived skill level meaning the conditions for flow are not present in these circumstances. Effort, ability, and luck are attributions contained within Seligman's dimension 'personal-external'. Confidence, and persistence relate to student inclination to cross those boundaries that separate the mathematically known from the mathematically unknown (black arrows in Figure 1 Diagram 1 (a) and (b)). Persistence is a characteristic related to the temporary nature of failure perceived by optimistic children, and confidence would be expected to relate to the self-sustaining nature of optimism (Seligman, 1995). The following research questions are posed: (a) Are the post-lesson video-stimulated student interviews from the Learners' Perspective Study sufficiently rich to provide indicators of student resilience; and (b) Is resilience associated with the student inclination to spontaneously pursue novel mathematical ideas?

Table 1
Seligman's (1995) Three Dimensions of Optimism Used as Indicators of Resilience

Dimension	Indicators of Lack of Resilience		Indicators of Resilience	
	Success.	*Failure*	*Success*	*Failure*
Permanent-Temporary	Temporary	Permanent	Permanent	Temporary
Pervasive-Specific	Specific	Generalised	Generalised	Specific
Personal-External	Attributed to external factors	Attributed to personal factors	Attributed to personal factors	Attributed to external factors

Study Design

A purposeful sample of 9 students was selected from the 86 students I interviewed (Australian Schools 1-4 (A1-4) and USA School 1 (US1)) and 2 students from USA School 3 (US3). This data set forms part of the Learners' Perspective international study of the classrooms of teachers seen by their school community to 'display good teaching practice'. Approximately 10 successive lessons were studied in each class using three video cameras that operated simultaneously in the classroom to 'capture' the actions of the: (a) whole class; (b) teacher; and (c) pair of focus students (different students each lesson). After the lesson, video-stimulated student interviews were conducted. Clarke (2001) explains the methodology in more detail. The purposeful sample was selected to: (a) illustrate the usefulness of the interviews for studying resilience; and (b) study associations between student inclination to pursue novel mathematical ideas and resilience. It included all of the students who had explicitly created or interconnected concepts during the research period (4 male students and 1 female student) and four students whose 'ways of working' suggested they were unlikely to participate in the spontaneous pursuit of novel mathematical ideas. Explicit demonstration of concept creation or interconnection during the lesson was used to operationalise "inclination to pursue novel mathematical ideas". The decision to restrict the data set to students I had interviewed was based upon the existence of prior analysis that informed the present study. This analysis identified those students (4 male students) who had explicitly created concepts or interconnected concepts (e.g., Williams, 2001, 2002b, submitted). The decision to extend the data set to include two students from US3 was based on the absence of a female student who had explicitly pursued novel mathematical ideas in the original data set.

Three of the five students who demonstrated concept creation or interconnection did so on more than one occasion. Kerri (US3) and Leon (A1), who both achieved excellent grades in mathematics, created concepts on at least two occasions, and Pepe (A1) who achieved above average grades, interconnected concepts on one occasion and created concepts in collaboration with Leon on another occasion. Eden (A2), who created concepts on one occasion, gained average grades in the year of the study even though he achieved 13 for problem solving on a national competition (average Year 8 problem solving score: 3). Dean (A4), who interconnected concepts on one occasion, struggled to achieve average grades. The four students selected as unlikely to participate in the spontaneous processes of concept creation or concept interconnection included two male students (Darius A2 and Jason A3) and two female students (Lara A1 and Sally A4). Darius (who achieved high grades) was interested in 'how' but not 'why' when solving mathematical problems, and Jason (who achieved low grades) focused on off-task interactions with his peers rather than his work. Lara and Sally (who achieved high grades) focused on the mathematics presented by the teacher; Sally was task-centred and Lara ego-centred (Nicholls, 1983). Sally worked at a faster pace than most class member and continually relied upon the teacher to check the correctness of her progress or assist when she had difficulties. To 'feel brave' Lara volunteered to answer questions she was uncertain about when no one had volunteered:

> It is kind of nerve wracking … you're like ohhh 'they're going to laugh at me … hope they don't think I'm stupid' … it sort of makes me feel more *brave* … I think 'hey I'm good for doing this'

Indicators of student orientation along the three dimensions of optimism are now discussed. Indicators along the permanent-temporary, and internal-external dimensions related to student: (a) judgements about themselves and their progress; (b) descriptions of how they learnt; and (c) reflections about successes and failures. Indicators along the pervasive-specific dimension related to whether students saw successes or failures as specific to the situation or as generalised characteristic of themselves (or others).

Results

This section is to be read in conjunction with Tables 1 and 2. It provides examples of student comments used as indicators of optimism. Table 2 Column 1 lists each student, and whether they pursued novel ideas (shaded region), and the dimensions of optimism (unshaded). Columns 2 and 3 (lack of resilience), and Columns 4 and 5 (resilience), display indicators along each of the dimensions of optimism for each student. When Kerri was asked how she figured out a novel idea she responded: "I don't know. I guess I'm smart". She attributed success to the personal characteristic: 'being smart' (optimistic along pervasive-specific and external-personal). Leon described his collaboration with Pepe demonstrating he perceived success as permanent and resulting from effort (optimistic along permanent-temporary and personal-specific):

> … we work really really well together 'cause n- it's not just one of us doing all of the work we both always work it out.

Eden qualified his average results by constraining his 'failure' to achieve high grades to his membership of 'this class' (optimistic on pervasive-specific):

Well in this class I think I am pretty much average … because I get pretty average scores… there is no way to explain.

Table 2

Associations Between Inclination to Pursue Novel Mathematical Ideas and Resilience

Name; Concept Creation** or Concept Interconnection* Dimensions of Optimism	Indicators of Lack of Resilience		Indicators of Resilience	
	Success (temp, spec, ext)	Failure (perm, perv, pers)	Success (perm, perv, pers)	Failure (temp, spec, ext)
Kerri **				
Perm-Temp			√	√
Perv-Spec			√	√
Personal-External			√	√
Leon **				
Perm-Temp			√	√
Perv-Spec			√	√
Pers-Ext			√	√
Pepe **				
Perm-Temp			√	√
Perv-Spec			√	
Pers-Ext			√	
Eden **				
Perm-Temp			√	√
Perv-Spec				√
Pers-Ext			√	
Dean *				
Perm-Temp			√	√
Perv-Spec			√	√
Pers-Ext			√	√
Darius				
Perm-Temp			√	√
Perv-Spec			√	√
Pers-Ext			√	
Sally				

Perm-Temp	√			√
Perv-Spec				
Pers-Ext	√		√	
Lara				
Perm-Temp				√
Perv-Spec		√	√	
Pers-Ext			√	
Jason				
Perm-Temp		√		√
Perv-Spec		√		
Pers-Ext	√	√		√

Key: Permanent: Perm; Temporary: Temp; Pervasive: Perv; Specific: Spec; Personal: Pers; External: Ext;

Darius demonstrated optimism along the permanent-temporary and personal-external dimensions: "… keep on trying until you get the system of it". Dean demonstrated optimism on the personal-external, and permanent-temporary dimensions by working out what he didn't understand while the teacher presented familiar work. Jason displayed contradictory indicators on the personal-external dimension; he judged himself 'good at maths' using his mother's perspective, and blamed the teacher's short pause time for his inability to answer questions.

Discussion and Conclusions

The student interviews from the Learners' Perspective Study provided rich indicators of student resilience or lack of resilience. The small number of students found pursuing novel mathematical ideas in this study should be considered in terms of the methodological approach; the cognitive activities of a different student pair were captured by the video and interview each lesson. Some students pursuing novel mathematical ideas may not have been identified. As all students found explicitly pursuing novel mathematical ideas showed indicators of resilience, the theoretical identification of a symbiotic interrelationship between student pursuit of novel mathematical ideas and student resilience has been empirically grounded. The student characteristics of resilience, and inclination to pursue novel mathematical ideas, appear to be mutually sustaining (overcoming a mathematical challenge conditions an optimistic orientation and an optimistic orientation increases student inclination to pursue the next challenge). The diversity of student achievement demonstrated by students who pursued novel ideas suggests these findings are relevant to pedagogical approaches in mainstream classrooms. Further research is required to study this phenomenon in more detail to find how to entice students who presently do not exhibit these characteristics into this cycle. This study raises many questions including: Would a focus on developing resilient students increase student inclination to pursue novel mathematical ideas? Would Darius have pursued novel mathematical ideas in a different classroom culture? Are the findings in this study a reflection of a general absence of spontaneous student activity in our mathematics classrooms? Why were there no females in the original data set? This study has interdisciplinary implications. To prevent or delay the onset of adolescent depression, Glover, Burns, Butler, and Patton (1998) are researching the effects of whole school programs designed to increase student resilience. By increasing the spontaneous and creative nature of student activity in mathematics classrooms, we may contribute to this initiative.

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References

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